Math 6462, Spectral Theory W22 suggested projects. Each student is required to give a 10-minute presentation on a topic of their choice at the end of the semester. The presentation should be on a subject related to spectral theory that was not covered in the course. Each talk should have an introduction and conclusion *and* contain a complete proof.[1] (See Halmos, Paul R.: "How to talk mathematics." Notices Amer. Math. Soc 21.3 (1974): 155-158 and Kra, Bryna: "Giving a Talk." Notices of the Amer. Math. Soc. 60.2 (2013), for some useful advice.)

You should choose a topic and inform me of your choice by mid-February. The topics will be given out on the first come, first served, basis. That said, under certain circumstances two students may be allowed to cover the same topic. In this case the students will be required to give coordinated (distinct) talks, so that student # 1 introduces the topics and states the basics and student # 2 covers a more advanced material. (Each of the talks will be expected to contain a proof.)

Here is a list of possible topics. The list is not exhaustive and I am open to suggestions. Some of the topics are very broad, some are very advanced (hence you would be able to provide only a glimpse at the theory, but remember that some proof is a mandatory component of your talk) and some are very specific. Some of the topics are grouped into meta-topics so that a group of students can give linked presentations.

(1) C*-algebras as noncommutative topological spaces.
(2) Pontryagin duality for locally compact abelian groups.
(3) Hamburger moment problem (given $\lambda_n \in \mathbb{R}$, for $n \geq 0$, is there a Borel probability measure μ on $\mathbb{R}$ such that $\lambda_n = \int x^n \, d\mu(x)$ for all $n \geq 0$?)
(4) Peter–Weyl theorem.
(5) Fuglede's theorem (an operator on a Hilbert space commutes with a normal operator T iff it commutes with T^*).
(6) Unbounded self-adjoint operators.
   (a) Cayley transform.
   (b) Spectral theorem for unbounded self-adjoint operators.
   (c) Connections to physics.
(7) Banach spaces with few linear operators (Argyros–Haydon).
(8) Connections (of spectral theory) to Fourier analysis (and/or harmonic analysis).
(9) Index theory (K-theory, Atiyah–Singer).

[1] Note that for most topics it is impossible to give complete proofs. It is up to you to choose a representative statement whose proof is simple enough to be given in a short talk.