York University
Math 3260 W18 final exam
April 11, 2018

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Notes
1. Do not open this booklet until you are instructed to do so.
2. Have a photo ID and a sessional student card on your desk ready for checkup.
3. No calculators or other aids (books, smartphones, notes,...) are allowed.
4. Duration of the exam will be exactly 180 minutes.
5. Read all questions carefully, and answer all questions as they are stated. Marks will be awarded only for answering questions stated in the paper. No marks will be awarded for questions not asked in the paper, even if they sound very similar to questions asked in the paper.
6. Present your solutions in a clear and well-organized manner. Show all work unless otherwise specified.
7. Circle the correct answer whenever applicable. Enter your answer into space provided whenever applicable. Do not try guessing; marks may be deducted for wrong multiple choice answers.
8. If you do not have enough space on the page to answer a question, then you may continue on a page at the end. If you do this, then write “This question is continued on page X”. A failure to do so will result in your work not being graded.
9. Good luck! 😊
1. Find the critical path from $v$ to $w$ in the following activity network. Show all work.

![Diagram](image_url)
2. Suppose that $G$ is a planar 3-regular graph each of whose faces is is bounded by a 5-cycle or by a 6-cycle. Prove that $G$ must have exactly twelve faces each of which is bounded by a 5-cycle.
3. Is the following statement true or false? (Circle the correct answer.)

An edge $e$ of a connected graph $G$ is a bridge (i.e., $G - e$ is disconnected) if and only if every spanning tree of $G$ contains it.

Justify your claim.
4. For which values of $n \geq 3$ is there a simple Hamiltonian graph $G$ with $n$ vertices such that $\deg(v) < n/2$ for all vertices $v$?

Clearly state your answer here: 

Prove your claim.
5. The chromatic polynomial of a graph $G$ is the polynomial $P_G$ such that $P_G(k)$ is the number of different ways in which the vertices of $G$ can be $k$-coloured so that adjacent vertices have different colours. Find the following:

$$P_{C_3}(k) = \_, \quad P_{C_4}(k) = \_, \quad P_{C_5}(k) = \_, \quad P_{C_6}(k) = \_.$$ 

Show all work.
6. Suppose that $G = G(V_1, V_2)$ is a bipartite graph with at least one edge such that $\deg(v) \geq \deg(w)$ for every $v \in V_1$ and every $w \in V_2$. Prove that $G$ has a complete matching.
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